Data Visualization and Basic Statistical Testing

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View course video.



Course Overview

Course Objective

Provide a foundation in the basic statistical methods and principles necessary to understand, interpret, and communicate insights from data.

Course Structure

Lecture 1: Getting to Know Your Data: Types of Data and Descriptive Statistics
 Lecture 2: Sampling Concepts and Comparing Two Means
 Lecture 3: Linear Models and Correlation
 Lecture 4: Comparing Proportions and Measures of Association



Lecture Outline

Types of Data (qualitative vs. quantitative)

U Summarizing Data

Graphical Methods

Numerical Summary Measures

Confidence Intervals and P-Values





Research Question



Hypotheses



Study Design



Data Collection



Data Analysis



Evidence-Based Information





Research Question



Hypotheses



Study Design



Data Collection



Data Analysis

A clear and focused research question will guide the research process. Every decision regarding study design, data collection, and data analysis should be connected to your research question.



Evidence-Based Information





Research Question



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Data Analysis



Evidence-Based Information

A hypothesis is an attempt to answer your question with an explanation that can be tested. Hypothesis testing is central to the statistical methods we will cover in this course.





Research Question



Hypotheses



Study Design



Data Collection







Evidence-Based Information

Study design, data collection methods, and analytic techniques are selected to address your research question.





Research Question



Hypotheses



Study Design



Data Collection



Data Analysis



Evidence-Based Information

Results are interpreted and effectively communicated through text, tables, and figures. Data has been successfully translated into actionable insights.



Types of Data

Before you begin any analysis, ask yourself:

What kind of data am I working with?

Data type determines...

- How you display the data in a table or graph
- How you summarize the data, graphically and numerically
- What statistical methods you use to analyze the data







Categorical Data

Nominal Variables

Categories are not ordered. Two-level nominal variables are dichotomous.

Race		Hispanic		
1	White	0	No	
2	Black	1	Yes	
3	Asian			
4	Other			

Ordinal Variables

The order among the categories is important, but not the numerical values.

Education Level				
1	Less than high school			
2	Graduated high school			
3	Some college or associate			
4	College degree or higher			



Numerical Data



Discrete Variables

- Order and magnitude are important
- Usually integers and counts



Continuous Variables

- Order and magnitude are still important
- Can take on any continuous value (not restricted to be integers)



Grouping Continuous Data

Continuous measurements can be transformed into ordinal or dichotomous ones.

Examples

- BMI categorized into underweight, normal, overweight, or obese
- Symptom score categorized into mild, moderate, or severe
- Age categorized as younger vs. older

Note: Putting data into buckets leads to a loss of information, but it makes sense when clinically meaningful cut-points exist



Bar Plots



- Display the frequency distribution for **nominal** or **ordinal** data
 - X-Axis displays categories
 - **Y-Axis** displays N or % of cases in each category (should always start at zero)
- Bars should have equal width and be separate from each other so as not to imply continuity



Histograms



- Display the frequency distribution for **discrete** or **continuous** data
- Similar to bar plots except...
 - X-Axis displays a range of values (instead of distinct categories)
 - **Y-Axis** displays N or % of cases in each interval/group of values
 - Bars are not separated from each other, since we assume "continuity" on the x-axis



Box Plots



The "Box": Represents the inter-quartile range (IQR), or middle 50% of the data

- Median: Marks the mid-point of the data, where half the values are greater than or equal to this value and half are less than or equal to this value
- **Upper Quartile:** 75th percentile (75% of scores fall below the upper quartile)
- Lower Quartile: 25th percentile (25% of scores fall below the upper quartile)



Box Plots



The "Whiskers": The definition of whiskers can vary. They may represent...

- The full extent of the data (maximum and minimum data points).
- The most extreme observations that are within 1.5*IQR.

Outliers are plotted outside the whiskers.



Box Plots



Box plots are useful for visualizing the distribution of variables.

- Presentation can be vertical or horizontal
- Length of the box indicates variation in percent agreement
 - For example, Morocco (green) and USA (gray) have less variation (shorter box
 → high % agreement) in life expectancy relative to Somalia (orange) and Mexico (purple)



Two-Way Scatter Plots



Scatter plots are used to show the relationship between two quantitative variables.

- Age vs. forced expiratory volume (FEV)
- Flow cytometry vs. image analysis (%)

Each point on graph represents a pair of values.

- x-axis is the scale for 1st variable (or independent variable)
- y-axis is the scale for 2nd variable (or dependent variable)



Two-Way Scatter Plot



Strength of relationship is measured by how close points are to the x=y trend line.

- Close to line \rightarrow strong correlation
- Further from line \rightarrow weak correlation

Things to remember:

- If scale for 1st & 2nd variable are the same, then use same range on the x-axis and y-axis.
- If plotting 1st variable against more than one variable, keep axis of 1st variable consistent in all graphs.



Numerical Summary Measures



Measures of central tendency or location

• Mean, median, and mode

Measures of dispersion

- Quantify the variation among the values
- Range, interquartile range, standard deviation and variance



Numerical Summary Measures

Natural way to convey information

- "On average most people exercise 3-4 times per week"
- "Most people would say that we have had a hot summer"

Where is the "center" of the data?

- Mean \rightarrow Average observation
- Median \rightarrow Middle observation
- Mode \rightarrow Most frequently occurring value



Measures of Central Tendency: Mean

- Calculated by summing all values and dividing by the number of observations
 - Takes the magnitude of each value into account
- Most commonly used measure of central tendency
 - Used for discrete and continuous data
 - In general not appropriate for categorical data
 - Exception dichotomous data. Assume cases alive have value of 1 and cases died value of 0.
 - Mean = proportion of cases with value of 1 = proportion of cases alive



Measures of Central Tendency: Mean

The mean is influenced by extreme values (outliers)

What to do in this case? Can we just delete these outliers?

No, we need measures that are not sensitive to outliers. One such measure is the median.



Measures of Central Tendency: Median

• Defined as the 50th percentile

- Rank observations from smallest to largest
- Half of the values are \geq median and other half are \leq median
- Does not take the magnitude of each value into account, just the position
- Equal to "center" of the ranked values
 - If odd number of observations \rightarrow middle value
 - If even number of observations \rightarrow average of 2 middle values
- Used for ordinal, discrete, and continuous data



Measures of Central Tendency: Mode

- Defined as the value or observation that occurs most frequently
- Used for all types of data

		Post-Treatment Condition	n	%
	1	Much improved	9	13%
Mode \rightarrow	2	Slightly improved	28	39%
	3	Stays the same	16	23%
	4	Slightly worse	12	17%
	5	Much worse	6	8%
		Total	71	100%



Example: Forced Expiratory Volume (FEV)

Order	FEV
1	2.15
2	2.25
3	2.30
4	2.60
5	2.68
6	2.75
7	2.82
8	2.85
9	3.00
10	3.38
11	3.50
12	4.02
13	4.05
Sum	38.35

FEV Data

- 13 subjects
- Values are ordered from smallest to largest

Mean FEV

- N = 13
- Sum = 38.35
- Mean = (38.35 / 13) = 2.95



Example: Forced Expiratory Volume (FEV)

Order	FEV
1	2.15
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12	4.02
13	4.05
Sum	38.35

Median FEV

- Order the data (smallest \rightarrow largest)
- With 13 observations, median is the value of order = (13+1) / 2 = 7
- Median = 2.82

What is the Mode?

• All FEV values are unique \rightarrow no mode

What if we have an even number of observations?



Example: Forced Expiratory Volume (FEV)

Order	FEV
1	2.15
2	2.25
3	2.30
4	2.60
5	2.68
6	2.75
7	2.82
8	2.85
9	3.00
10	3.38
11	3.50
12	4.02

Median FEV

- Order the data
- With 12 observations, median is average of 6th and 7th values

Median = (2.75 + 2.82) / 2 Median = 2.785



Best measure to report?

- Depends on the type of data and the way in which the values are distributed
 - If data is symmetric and unimodal (i.e., only 1 peak), then mean, median, and mode should all be roughly equal
 - If data is skewed, then the mean is sensitive to extreme values → the median might be more appropriate
- What do I mean by skewed data?



Symmetric vs. Skewed



Skewed to right

- Mostly low values
- Median might be more appropriate

Symmetric

 Mean and median would be about the same

Skewed to left

4

- Mostly high values
- Median might be more appropriate

6

8

10



Example of skewed distribution: Household income





Example of skewed distribution: Household income





Important to use BOTH numerical measures & graphical methods

Example: Distribution of birth weight among infants (<1750 g)

Mean=1,173 Median=1,140 Mean ≈ Median, so we expect a symmetric distribution

What do you think about the histogram?



Weight (grams)



Important to use BOTH numerical measures & graphical methods

Example: Distribution of birth weight among infants (<1750 g)

Mean=1,173 Median=1,140 Mean ≈ Median, so we expect a symmetric distribution

What do you think about the histogram?

- Bimodal distribution
- Low and high weight appear to have different centers





Measures of Dispersion: Range

Range: Difference in smallest and largest values

- Easy to compute but <u>not</u> very useful because...
 - Uses only extreme data to calculate
 - Highly sensitive to very small and large values

An alternative...

Interquartile Range (IQR): Difference in 25th and 75th percentiles

- Includes middle 50% of the data
- Less sensitive to very small and large values



Measures of Dispersion: Variance

Sample Variance



Variance

- Indicates... average degree to which each point differs from the mean
- Calculated as... average of squared distance between observed value and mean value

Standard Deviation

- Indicates... how spread out a group of numbers is from the mean
- Calculated as... the square root of the variance
- Standard deviation is used because it has the same units of measurement as the mean



Sample Standard Deviation

 $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$

Example: Calculating Variance & SD

FEV	(x _i – x)		
2.30	-0.65		
2.15	-0.80		
3.50	0.55		
2.60	-0.35		
2.75	-0.20		
2.82	-0.13		
4.05	1.10		
2.25	-0.70		
2.68	-0.27		
3.00	0.05		
4.02	1.07		
2.85	-0.10		
3.38	0.43		
Total			

Step 1: Calculate difference between each observation and the mean.

N = 13 Mean = 2.95



Example: Calculating Variance & SD

FEV	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$	
2.30	-0.65	0.423	
2.15	-0.80	0.640	
3.50	0.55	0.303	
2.60	-0.35	0.123	
2.75	-0.20	0.040	
2.82	-0.13	0.169	
4.05	1.10	1.210	
2.25	-0.70	0.490	
2.68	-0.27	0.073	
3.00	0.05	0.003	
4.02	1.07	1.145	
2.85	-0.10	0.010	
3.38	0.43	0.185	
Total	0.0	4.66	

Step 2: Square the difference between each observation and the mean. Sum.

Step 3: Divide by number of observations minus 1.

Variance = 4.66 / (13-1) = 0.388

Step 4: Calculate standard deviation as square root of the variance.

Standard = $\sqrt{0.388}$ Deviation = 0.623



Summarizing Data in SPSS (Graphical)

SPSS: Analyze > Descriptive Statistics > Frequencies



	Frequencies: Charts
Chart Type	
O None	
 Bar cha 	arts
🔵 Pie cha	irts
🔵 Histog	rams:
Sho	w normal curve on histogram
Chart Value	es ncies O Percentages
?	Cancel Continue



Summarizing Data in SPSS (Numerical)

SPSS: Analyze > Descriptive Statistics > Descriptives





Presenting Numerical Summaries

	Variables	Treatment (N=628)	Control (N=372)		
	Age				
Continuous Variables:	N (Nmiss)	628 (0)	372 (0)	\ .	
Mean \pm SD	Mean ± SD	41.6±7.6	48.8±7.4		ortant:
Median (IQR)	Min-Max	29.0–59.0	29.0–61.0		ays present
	Median (IQR)	40.5 (35.5–48.0)	50.0 (44.5–55.0)	1.	The number of
Categorical Variables:	Gender (%)				observations (N) in
	Female	414 (65.9)	173 (46.5)		the sample
IN (%)	Male	214 (34.1)	199 (53.5)	2.	The number of
	Weight				observations
	N (Nmiss)	628 (0)	370 (2)		missing for each
	Mean ± SD	148.8±26.4	157.3±28.3		variable
	Min-Max	87.0–243.0	71.0–250.0		
	Median (IQR)	146.0 (129.5–165.0)	155.0 (137.0–175.0)		

N, number of non-missing values; Nmiss, number of missing values; SD, standard deviation; IQR, interquartile range.



Questions?





What's next?

Now that we have mean and standard deviation, how do we know if results are "important" or "significant"?

Two common quantities are reported:

- Confidence intervals
- P-values

Why do we calculate and report these?

• To draw "inferences" about a population mean from our sample



Calculating a Confidence Interval (CI)

Confidence intervals indicate the precision of the sample mean with n subjects (reflects variability in sampling)

• Lower Limit = mean $-\left(Z_{critical} * \frac{SD}{\sqrt{n}}\right)$

• Upper Limit=
$$mean + \left(Z_{critical} * \frac{SD}{\sqrt{n}}\right)$$

$Z_{critical}$ (also denoted $Z_{1-\alpha/2}$)

- Measures the number of standard errors to be added and subtracted from the mean in order to achieve a desired confidence level
- Equal to 1.96 for 95% confidence interval
- Obtained from a "reference" distribution (more on this next lecture...)



Example: Calculating a Confidence Interval



95% Lower Limit= $2.03 - 1.96 * 0.51/\sqrt{309} = 1.97$ 95% Upper Limit= $2.03 + 1.96 * 0.51/\sqrt{309} = 2.09$

We'll come back to the interpretation...



What does a confidence interval mean?

True Population Mean = 2.5



A fundamental task of biostatistics is to analyze <u>samples</u> in order to make inferences about the <u>population</u> from which the samples were drawn.

A confidence interval tells us how confident we can be that the results of a study reflect what we would expect to find if it were possible to survey the entire population.



What does a confidence interval mean?

True Population Mean = 2.5



So, a 95% confidence interval is a range of values that we can be 95% certain contains the true mean of the population.

In practice...

- Repeat an experiment 20 times
- Calculate the 95% CIs for each experiment
- Expect 95% of the intervals to contain the true population mean (≥ 19 of the intervals)

Confidence Intervals & P-Values

Example: Calculating a Confidence Interval



Interpretation:

Based on our sample data, we are 95% confident that the true mean FEV in the population is between 1.97 and 2.09.

95% Lower Limit= $2.03 - 1.96 * 0.51/\sqrt{309} = 1.97$

95% Upper Limit= $2.03 + 1.96 * 0.51/\sqrt{309} = 2.09$



P-Values

- Another common quantity reported is the p-value
- Defined as the probability of observing a test statistic more extreme than the observed value by chance
- Values range between 0-1
 - Small $p \rightarrow$ result not likely due to chance
 - Large $p \rightarrow$ result is likely due to chance
- Tied to concept of hypothesis testing



P-Values & Hypothesis Testing

Hypothesis Testing

- Purpose: examine whether a difference (or an effect) is present or not by using statistical tests
- Decision:
 - "Yes, there is a significant difference..."
 - "No, there is not a significant difference ..."
- P-values are tied to "significance" → p < 0.05 is a commonly used threshold for significance



3

Process of Hypothesis Testing

Define null (H_0) and alternative (H_1) hypotheses.

Example: Means of 2 groups: equal (H_0) vs. not equal (H_1) Calculate the probability of observing a value \geq **your test statistic** if the means are equal (i.e., H₀ is true) \rightarrow

p-value

Determine how different the observed data is from the null hypothesis \rightarrow

2

calculate your test statistic

Decide whether or not to reject the null hypothesis that the means are equal

4



Next Class

- To fully understand confidence intervals and p-values, we need to take a step back and briefly discuss a few concepts:
 - Sampling
 - Normal Distribution
 - Central Limit Theorem
- This will lead us into the comparison of two means



Questions?

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